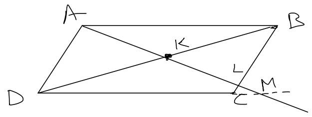
Q> Point K lies on diagonal BD of parallelogram ABCD. AK intersects lines BC and CD at L and M respectively. Prove that $AK^2 = LK \cdot KM$.



Ans:- $\Delta AKD \approx \Delta BKL \Rightarrow \frac{AK}{KL} = \frac{DK}{BK} = \frac{AD}{BL}$

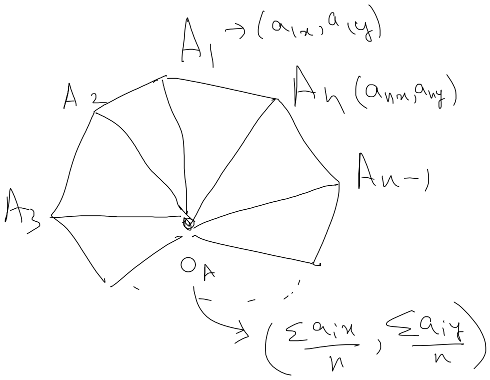
$\Delta DKM \approx \Delta AKB \Rightarrow \frac{KM}{AK} = \frac{DM}{AB} = \frac{KD}{BK}$

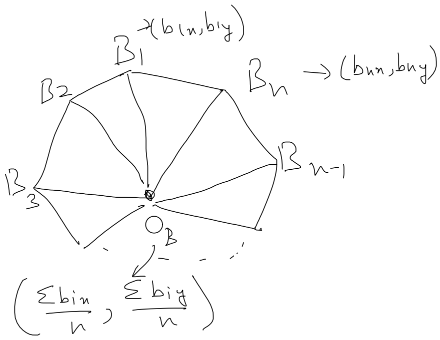
$\Rightarrow \frac{AK}{KL} = \frac{KM}{AK} \Rightarrow AK^2 = KL \cdot KM$

Similarity in polygons:-

Polygons $A_1A_2 \cdots A_n$ and $B_1B_2 \cdots B_n$ are similar if $A_1A_2 : A_2A_3 : \cdots : A_nA_1 = B_1B_2 : B_2B_3 : \cdots : B_nB_1$ and all the angles $\angle A_i = \angle B_i$.

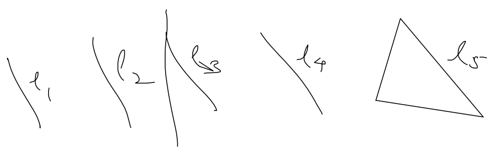
•> The same proportion holds for the diagonals as well





Q> It is known that from a set of 5 line segment it is possible to form 4 different right triangles. Find the square of the ratio of the largest segment to the smallest.

$l_1 < l_2 < l_3 < l_4 < l_5$



$a^2 + b^2 = c^2$

Ans:- Case 1

$l_5^2 = l_3^2 + l_2^2 = l_4^2 + l_1^2$

Case 2

$l_5^2 = l_4^2 + l_2^2 = l_3^2 + l_1^2$ $\longrightarrow$ so not possible

$l_4 = l_3^2 + l_2^2$, $l_3^2 + l_1^2$, $l_2^2 + l_1^2$ } none of them are same so l_4 doesn't repeat

l_5 repeats $\Leftarrow$

$l_5^2 = l_3^2 + l_2^2 = l_4^2 + l_1^2$

$\Rightarrow l_5^2 = 2l_2^2 + l_1^2$

$l_3^2 = l_2^2 + l_1^2$

$l_4^2 = l_3^2 + l_1^2$ or $\cancel{l_2^2 + l_1^2}$

$l_5^2 = l_3^2 + 2l_1^2$

$= l_2^2 + 3l_1^2$ .. (as $l_3^2 = l_2^2 + l_1^2$)

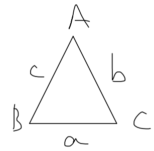
$\Rightarrow 2l_2^2 + l_1^2 = l_2^2 + 3l_1^2$

$\Rightarrow l_2^2 = 2l_1^2 \longrightarrow \Rightarrow l_5^2 = 2l_2^2 + l_1^2 = 4l_1^2 + l_1^2 = 5l_1^2$

$\Rightarrow \dfrac{l_5^2}{l_1^2} = 5$

Q) In ΔABC, angles α and β are related as $3\alpha + 2\beta = 180°$.

Prove that $a^2 + bc = c^2$ (let third angle be γ)



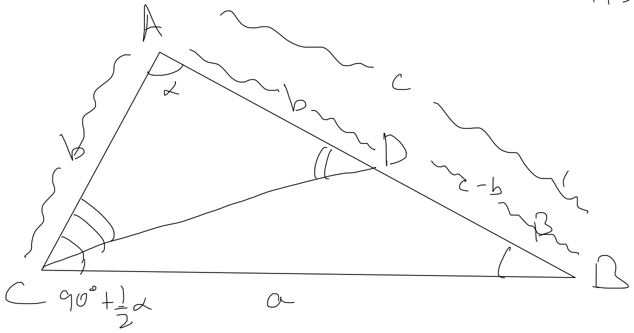
$2(\alpha+\beta) + \alpha = 180° \Rightarrow \alpha + \beta = 90° - \frac{1}{2}\alpha$

$\gamma = 180° - (\alpha+\beta) = 90° + \frac{1}{2}\alpha$

$\gamma > \alpha + \beta$

Ans:-

$AC = AD$



We want to show,

$a^2 + bc = c^2$

$\Rightarrow a^2 = c^2 - bc = c(c-b)$

$\Rightarrow \dfrac{a}{c-b} = \dfrac{c}{a}$

In ΔABC and ΔCBD, β is common angle

$\angle CDB = 180° - (90° - \frac{1}{2}\alpha) = 90° + \frac{1}{2}\alpha = \angle ACB \Rightarrow \Delta ABC \simeq \Delta CBD$

$\Rightarrow \dfrac{a}{c-b} = \dfrac{c}{a} \Rightarrow a^2 + bc = c^2$

HomeWork:-

Q) The length of two sides of a triangle are equal to a and the third side length is b. Find the length of radius of

Q) The length of two sides of a triangle are equal to a and the third side length is b. Find the length of radius of circumcircle of this triangle.